Evaluate the following integrals.

SCORE: _____ / 60 PTS

[a] $\int_1^3 \arctan\sqrt{x}\,dx$

(4) $u=\sqrt{x} \rightarrow x=u^2$

$dx=2u\,du$

(4) $\int_1^{\sqrt{3}} 2u\arctan u\,du$

(3)

$\arctan u$	$2u$
$\frac{1}{1+u^2}$	$\Rightarrow u^2$

$= u^2\arctan u\Big|_1^{\sqrt{3}} - \int_1^{\sqrt{3}} \frac{u^2}{1+u^2}\,du$ (3) (3)

$= 3\left(\frac{\pi}{3}\right) - \frac{\pi}{4} - \int_1^{\sqrt{3}} \left(1-\frac{1}{1+u^2}\right)du$ (3)

$= \frac{3\pi}{4} - (u - \arctan u)\Big|_1^{\sqrt{3}}$ (4)

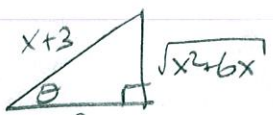
$= \frac{3\pi}{4} - \left(\sqrt{3} - \frac{\pi}{3} - \left(1-\frac{\pi}{4}\right)\right)$ (3)

$= \frac{3\pi}{4} - \sqrt{3} + \frac{\pi}{3} + 1 - \frac{\pi}{4}$

$= \frac{5\pi}{6} + 1 - \sqrt{3}$ (3)

[b] $\int \sqrt{x^2+6x}\,dx$

$= \int \sqrt{(x+3)^2-9}\,dx$



$x+3=3\sec\theta$

$dx=3\sec\theta\tan\theta\,d\theta$

$= \int (3\tan\theta)(3\sec\theta\tan\theta)\,d\theta$

$= 9\int \tan^2\theta\sec\theta\,d\theta$

$= 9\int (\sec^2\theta-1)\sec\theta\,d\theta$

$= 9\int (\sec^3\theta - \sec\theta)\,d\theta$

$= 9\left(\frac{1}{2}(\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta|) - \ln|\sec\theta+\tan\theta|\right) + C$

$= \frac{9}{2}\sec\theta\tan\theta - \frac{9}{2}\ln|\sec\theta+\tan\theta| + C$

$= \frac{9}{2}\cdot\frac{x+3}{3}\cdot\frac{\sqrt{x^2+6x}}{3} - \frac{9}{2}\ln\left|\frac{x+3}{3} + \frac{\sqrt{x^2+6x}}{3}\right| + C$

$= \frac{1}{2}(x+3)\sqrt{x^2+6x} - \frac{9}{2}\ln\left|x+3+\sqrt{x^2+6x}\right| + C$

$2\frac{1}{2}$ EACH

Evaluate the following integrals. **SCORE: _____ / 45 PTS**

[a] $\int_{-4}^{4} \frac{A\ 32}{x^4-16}\,dx$

$$= \int_{-4}^{-2} \frac{32}{x^4-16}\,dx + \int_{-2}^{0} \frac{32}{x^4-16}\,dx + \int_{0}^{2} \frac{32}{x^4-16}\,dx + \int_{2}^{4} \frac{32}{x^4-16}\,dx \quad (3)$$

$$\int \frac{32}{x^4-16}\,dx = \int \frac{32}{(x^2-4)(x^2+4)}\,dx$$

$$= \int \frac{32}{(x+2)(x-2)(x^2+4)}\,dx$$

$$= \int \left(\frac{A}{x+2} + \frac{B}{x-2} + \frac{C(2x)+D}{x^2+4}\right)dx \quad (3)$$

$$32 = A(x-2)(x^2+4) + B(x+2)(x^2+4) + C(2x)(x+2)(x-2) + D(x+2)(x-2) \quad (3)$$

$x=2$: $32 = B(4)(8) \rightarrow B = 1$

$x=-2$: $32 = A(-4)(8) \rightarrow A = -1$

COEF OF x^3: $0 = A+B+2C \rightarrow C = -\frac{1}{2}(A+B) = 0$

$x=0$: $32 = -8A+8B-4D \rightarrow 32 = 16-4D \rightarrow D = -4$

SANITY CHECK: $x=3$ $\quad \frac{32}{3^4-16} \stackrel{?}{=} \frac{-1}{5} + \frac{1}{1} - \frac{4}{13}$

$$\frac{32}{65} = \frac{-13+65-20}{65} \checkmark$$

[b] $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^2 \tan x\,dx$

$x^2 \tan x$ CONTINUOUS ON $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (2)

$(-x)^2 \tan(-x) = -x^2 \tan x$ (3) ODD (2)

INTEGRAL $= 0$ (3)

$$\int_0^2 \frac{32}{x^4-16}\,dx = \lim_{N\to 2^-} \int_0^N \left(-\frac{4}{x+2} + \frac{4}{x-2} - \frac{4}{x^2+4}\right) dx$$

$$= \lim_{N\to 2^-} \left(-\ln|x+2| + \ln|x-2| - 2\tan^{-1}\frac{x}{2}\right)\Big|_0^N$$

$$= \lim_{N\to 2^-} \left(-\ln|N+2| + \ln|N-2| - 2\tan^{-1}\frac{N}{2} - \ln 2 + \ln 2 - 0\right)$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$-\ln 4 \qquad -\infty \qquad -\frac{\pi}{2}$$

DIVERGES

Find $\int \frac{3-x^4}{x^3+2x^2+x}\,dx$.

SCORE: ____ / 30 PTS

$$= \int \left(-x+2+\frac{-3x^2-2x+3}{x(x+1)^2}\right)dx$$

$$= \int \left(-x+2+\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}\right)dx$$

$$x^3+2x^2+x \;\overline{)\; -x^4 \qquad\qquad +3}$$

$-x+2$

$-x^4-2x^3-x$

$2x^3+x$

$2x^3+4x^2+2x$

$-3x^2-2x+3$

$$-3x^2-2x+3 = A(x+1)^2+Bx(x+1)+Cx$$

$x=0$: $\quad 3 = A$

$x=-1$: $\quad 2 = -C \rightarrow C=-2$

COEF OF x^2: $\quad -3 = A+B \rightarrow B=-3-A=-6$

SANITY CHECK $x=2$: $\quad \frac{-13}{2(9)} \stackrel{?}{=} \frac{3}{2}+\frac{-6}{3}+\frac{-2}{9}$

$$\frac{-13}{18} = \frac{27-36-4}{18} \;\checkmark$$

$$= \int \left(-x+2+\frac{3}{x}-\frac{6}{x+1}-\frac{2}{(x+1)^2}\right)dx$$

$$= -\tfrac{1}{2}x^2+2x+3\ln|x|-6\ln|x+1|+\frac{2}{x+1}+C$$

(2½ EACH)

Determine if $\int_0^1 \frac{4+\sin x}{\sqrt{x}}\,dx$ converges or diverges. Justify your answer properly.

SCORE: ____ / 15 PTS

$-1 \le \sin x \le 1$

$3 \le 4+\sin x \le 5$

$0 \le \frac{4+\sin x}{\sqrt{x}} \le \frac{5}{\sqrt{x}}$ ON $(0,1)$

(3 EACH)

$\int_0^1 \frac{5}{\sqrt{x}}\,dx$ CONVERGES $(p=\frac{1}{2}<1)$

SO $\int_0^1 \frac{4+\sin x}{\sqrt{x}}\,dx$ CONVERGES